

ON THE MATCHING CONDITIONS FOR THE COLLAPSING CYLINDER

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Abstract

We review the matching conditions for a collapsing anisotropic cylindrical perfect fluid, recently discussed in the literature (2005 *Class. Quantum Grav.* **22** 2407). It is shown that radial pressure vanishes on the surface of the cylinder, contrary to what is asserted in that reference. The origin of this discrepancy is to be found in a mistake made in one step of the calculations. Some comments about the relevance of this result in relation to the momentum of Einstein–Rosen waves are presented.

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1 Introduction

In a recent paper [1], using the Darmois conditions [2], the matching of a general collapsing anisotropic cylindrical perfect fluid to the Einstein–Rosen (E–R) spacetime [3] is studied in detail.

As the main result of that study, it appears that the radial pressure does not vanish at the boundary surface of the matter distribution. The physical interpretation of this surface pressure is justified through the continuity of the radial flux of momentum across the boundary surface.

However, as we shall see in this note, such a result is incorrect. Indeed, a careful review of the calculation process indicates a mistake in one of the final steps. Once this mistake is corrected, the final result is the vanishing of the radial pressure on the boundary surface. This implies that any local pressure effect, predicted by a local characterization of the energy (momentum) flux of gravitational energy of the E–R waves, by means of pseudo-tensors (see e.g. [4] and references therein), will not be observed.

Let us show how this comes about.

2 The collapsing perfect fluid cylinder and the Einstein–Rosen metric

We consider a collapsing cylinder filled with anisotropic non-dissipative fluid bounded by a cylindrical surface Σ .

We assume the general time dependent cylindrically symmetric metric

$$ds_-^2 = -A^2(dt^2 - dr^2) + B^2dz^2 + C^2d\phi^2, \quad (1)$$

where A , B and C are functions of t and r and we choose the fluid to be comoving in this coordinate system.

For the exterior vacuum spacetime of the cylindrical surface Σ we take the metric in Einstein-Rosen coordinates [3],

$$ds_+^2 = -e^{2(\gamma-\psi)}(dT^2 - dR^2) + e^{2\psi}dz^2 + e^{-2\psi}R^2d\phi^2, \quad (2)$$

where γ and ψ are functions of T and R satisfying the field equations $R_{\alpha\beta} = 0$.

3 Junction conditions and their consequences

The necessary and sufficient conditions for a smooth matching, without a surface layer, are the Darmois junction conditions [2, 5], which require the continuity of the first and second fundamental forms across the boundary surface.

The equations of Σ may be written

$$f_- = r - r_\Sigma = 0, \quad (3)$$

$$f_+ = R - R_\Sigma(T) = 0, \quad (4)$$

where f_- refers to the spacetime interior of Σ and f_+ to the spacetime exterior, and r_Σ is a constant because Σ is a comoving surface forming the boundary of the fluid.

Using (3) in (1) we have for the metric on Σ

$$ds^2 \stackrel{\Sigma}{=} -d\tau^2 + B^2 dz^2 + C^2 d\phi^2, \quad (5)$$

where we define the time coordinate τ only on Σ by

$$d\tau \stackrel{\Sigma}{=} A dt, \quad (6)$$

and $\stackrel{\Sigma}{=}$ means that both sides of the equation are evaluated on Σ . We shall take $\xi^0 = \tau$, $\xi^2 = z$ and $\xi^3 = \phi$ as the parameters on Σ .

Then, the conditions on the interior and exterior metrics imposed by the continuity of the first fundamental form on Σ are (see [1] for details)

$$e^{\gamma-\psi} \left[1 - \left(\frac{dR}{dT} \right)^2 \right]^{1/2} dT \stackrel{\Sigma}{=} d\tau, \quad (7)$$

$$e^\psi \stackrel{\Sigma}{=} B, \quad (8)$$

$$e^{-\psi} R \stackrel{\Sigma}{=} C. \quad (9)$$

Next, differentiating (8) and (9) with respect to (6) and (7) we obtain

$$e^{2\psi-\gamma} \left[1 - \left(\frac{\dot{R}}{\dot{T}} \right)^2 \right]^{-1/2} \frac{d\psi_\Sigma}{dT} \stackrel{\Sigma}{=} \frac{B_{,t}}{A}, \quad (10)$$

$$e^{-\gamma} \left[1 - \left(\frac{\dot{R}}{\dot{T}} \right)^2 \right]^{-1/2} \left(\frac{\dot{R}}{\dot{T}} - R \frac{d\psi_\Sigma}{dT} \right) \stackrel{\Sigma}{=} \frac{C_{,t}}{A}. \quad (11)$$

where

$$\frac{d\psi_\Sigma}{dT} \stackrel{\Sigma}{=} \psi_{,T} + \psi_{,R} \frac{dR}{dT}, \quad (12)$$

and the dot denotes the derivative with respect to τ in the surface. Eqs.(10) and (11) are the corrected versions of (36) and (37) in [1].

Then proceeding as in [1], which involves the continuity of the second fundamental form, we obtain (in agreement with Israel's result [6])

$$P_r \stackrel{\Sigma}{=} 0. \quad (13)$$

The result (13) implies that the flux of momentum of the gravitational wave emerging from the cylinder will not produce any observable local pressure.

References

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